AIAA JOURNAL
Vol. 37, No. 5, May 1999

Single Finite States Modeling of Aerodynamic Forces Related to Structural Motions and Gusts

G. Pasinetti* and P. Mantegazza†
Politecnico di Milano, 20158 Milan, Italy

An integrated approach is presented for determining a single low-order, finite states, time-invariant approximation for the aerodynamic transfer matrices relating the generalized aerodynamic forces to small structural motions and gusts. The viability of such a formulation is justified in relation to a numerical solution of an unsteady linear potential flow, and a unified framework for the identification of the parameters of the aerodynamic system is presented. The implementation of the methodology thus developed adopts a linear least-square technique to determine a polynomial matrix approximation, which is subsequently transformed into a state-space model that is optimized and reduced to the lowest possible order by using a limited-memory, quasi-Newton minimization. The importance of a good fit of the aerodynamic response at low reduced frequencies is emphasized in relation to the need for correctly modeling the whole flight dynamics of a deformable aircraft. Some numerical results are included to demonstrate the efficiency of the proposed method.

Nomenclature

A, B, C, D = state matrices of a finite states approximation (as defined by the appropriate subscript)
C_{ae} = aeroelastic damping term
C_s = structural damping
c_p = pressure coefficient
$\boldsymbol{f}$ = generic generalized force vector
$\boldsymbol{f}_a$ = generalized unsteady aerodynamic force vector as defined by $\boldsymbol{f}$ equal to $q\boldsymbol{f}_a$
H = generic aerodynamic transfer matrix (as defined by the appropriate subscript)
j = imaginary unit $\sqrt{-1}$
K_{ae} = aeroelastic stiffness term
K_s = structural stiffness
k = harmonic reduced circular frequency, $(\omega l/V_\infty)$
l = aerodynamic reference length
M_{ae} = aeroelastic mass term
M_s = structural mass
M_∞ = freestream Mach number
N, D, R = without subscript a, numerator, denominator, and remainder of a left matrix polynomial parameterization of H; otherwise as defined earlier
P = generic aerodynamic impulse response matrix (as defined by the appropriate subscript)
p = complex reduced frequency, (sl/V_∞)
Q = generalized structural displacement shape functions (vibration modes)
q = freestream dynamic pressure, $(1/2\rho V_\infty^2)$
$\boldsymbol{q}$ = generalized structural degrees of freedom
s = complex circular frequency, i.e., Laplace variable, $(\sigma + j\omega)$
t = time
$\boldsymbol{u}$ = structural displacement field
V_∞ = freestream speed
v_g = gust velocity field component normal to the aircraft surface
$\boldsymbol{v}_g$ = gust velocity field
$\boldsymbol{x}$ = without subscripts, space coordinates vector; with subscripts, aerodynamic state vector
σ = without subscripts, real part of the complex frequency s
$\sigma_{\max}(X)$ = largest singular value of a matrix X
τ = nondimensional time $(V_\infty t/l)$
Φ = velocity potential shape functions
ϕ = velocity potential nodal parameters
φ = perturbation velocity potential, scaled by V_∞
ω = harmonic circular frequency; imaginary part of the complex frequency s

Subscripts

a = aerodynamics
e = elastic structure
f = fast dynamics partition of the aerodynamic state
g = gust
m = generic structural motion input to the aerodynamic
$q, \dot{q}, \ddot{q}$ = generalized structural displacements, velocities, and accelerations
s = slow dynamics partition of the aerodynamic state

Superscripts

$i, '$ = derivatives with respect to τ
$\cdot$ = derivatives with respect to t

Introduction

IN recent years the potentials and feasibility of actively controlled aeroelastic systems have been widely demonstrated. The push to adopt modern control techniques to design flutter suppression and load alleviation systems has revived the need to develop effective, reduced-order state-space realizations for the aerodynamic transfer matrices relating the generalized aerodynamic forces to small structural motions and gusts. Independent from any active control application, various types of such an approximation have already been in use for a long time, both for analog simulations and to translate flutter analyses into standard eigenproblems.[1–3] Moreover, the availability of a state-space approximation for the unsteady aerodynamics allows any linear aeroservoelastic system to be cast into a standard form. Thus, one can get rid of all of the specialized techniques adopted for the solution of the related stability and response problems and nevertheless gain all of the advantages by exploiting a host of easily available, standard, effective, general-purpose numerical procedures.

Analogous to what has happened in the control field, the still more common frequency approach to aeroelastic analyses has been dubbed *classical aeroelasticity*, whereas the term *modern*

Received Jan. 28, 1998; revision received Dec. 22, 1998; accepted for publication Dec. 23, 1998. Copyright © 1999 by G. Pasinetti and P. Mantegazza. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission.

*Graduate Engineer, Dipartimento di Ingegneria Aerospaziale, via La Masa, 34.
†Professor of Aeroelasticity, Dipartimento di Ingegneria Aerospaziale, via La Masa, 34. E-mail: mantegazza@aero.polimi.it.

aeroelasticity is reserved for the state-space formulation based on finite states aerodynamic approximations.[4,5] Whatever name is adopted, note that an effective and precise modeling of any aeroservoelastic system is strongly influenced by the difficulties related to an appropriate evaluation of the unsteady aerodynamic loads. Whereas direct nonlinear simulations based on the use of computational fluid dynamics (CFD) and structural finite elements are gaining momentum, the most common approach is still based on linear(ized) servostructural models coupled to linear(ized) aerodynamic forces related to the different trimmed steady flight conditions of interest.

Within such a framework, the unsteady aerodynamic formulations usually adopted are mostly based on the solution of linear integral equations related to harmonic boundary conditions from which the generalized unsteady aerodynamic forces are readily available in terms of frequency response (transfer) matrices for small structural motions and gusts. Such an approach is so deeply rooted that there is a tendency to determine the aerodynamic transfer matrices as well when linearizations are carried out numerically within a CFD formulation, so that a single aerodynamic interface can be established for all linear aeroservoelastic analyses. They can be written as

$$f_a = H_{am}(k, M_\infty)q + H_{ag}(k, M_\infty)\frac{v_g}{V_\infty} = [H_{am} \quad H_{ag}]\begin{Bmatrix} q \\ \dfrac{v_g}{V_\infty} \end{Bmatrix} \tag{1}$$

and because the matrices are obtained numerically, they are generally available at a discrete set of harmonic reduced frequencies $k = \omega l / V_\infty$.

Various methods have been developed to effectively identify a time-domain realization for Eq. (1)[6–14] with two distinct state-space approximations of $H_{am}(k, M_\infty)$ and $H_{ag}(k, M_\infty)$ having the same parametric structure. In the following, a more effective and unified approach for determining a single state-space approximation for all of the aerodynamic forces is presented and justified.

Single Finite States Approximation of Linear Aerodynamics

We now show how a numerical solution of a linearized unsteady aerodynamic formulation leads to a single finite states approximation of the aeroelastic response equations. To this end we begin by assuming a discretization of the structural response given by

$$M_s\ddot{q}(t) + C_s\dot{q}(t) + K_s q(t) = q f_a(t) + f(t) \tag{2}$$

where M_s, C_s, and K_s are the structural mass, damping, and stiffness matrices, f is any generic external forcing term, and q is an appropriate set of free generalized structural degrees of freedom associated with the displacement shape functions $Q(x)$, i.e., modeling the displacement field through a discretization of the type

$$u(x, t) = Q(x)q(t) \tag{3}$$

where $x = [x, y, z]^T$ are the space coordinates.

For simplicity we refer to an unsteady aerodynamic formulation based on a small perturbation velocity potential related to a slender body approximation, i.e., an aircraft flattened onto the xy and yz planes, with x the streamwise direction, but similar conclusions could be drawn starting from a different aerodynamic formulation. A linear compressible and isentropic flow around a slender body is then governed by the following set of equations in the perturbation velocity potential $V_\infty\varphi(x, \tau)$, where $\tau = (V_\infty/l)t$:

$$(M_\infty/l)^2\varphi_{\tau\tau} + 2\left(M_\infty^2/l\right)\varphi_{x\tau} - \left[\left(1 - M_\infty^2\right)\varphi_{xx} + \varphi_{yy} + \varphi_{zz}\right] = 0 \tag{4}$$

On the body surface σ, assuming $(\partial\varphi/\partial x)n_x \cong 0$ because of the slenderness, we can adopt the following boundary condition:

$$\varphi_n \cong \frac{\partial\varphi}{\partial y}n_y + \frac{\partial\varphi}{\partial z}n_z = N_q(x)q(\tau) + \frac{1}{l}N_{\dot{q}}(x)q'(\tau) + \frac{v_g(x, \tau)}{V_\infty} \tag{5a}$$

where $N_q(x)$ and $N_{\dot{q}}(x)$ are matrices, appropriately derived from $Q(x)$, defining the unit vector normal to the deformed moving body; and $v_g(x, \tau/V_\infty)$ represents a transverse gusts. At infinity we simply impose

$$\varphi_\infty = 0 \tag{5b}$$

whereas we must set

$$\Delta\varphi_x + (1/l)\Delta\varphi_\tau = 0 \tag{5c}$$

where Δ is the difference between the upper and lower side of the wake, to impose the no-load condition over any assigned thin wake trailing lifting bodies along the freestream, i.e., the Kutta condition. Finally, the linearized pressure coefficient is recovered with

$$c_p = -2[\varphi_x + (1/l)\varphi_\tau] \tag{6}$$

An approximate weak numerical solution of Eqs. (4) and (5) can then be searched by a weighted residual approach of the type

$$\int_\Omega W^T\left\{\left(\frac{M_\infty}{l}\right)^2\varphi_{\tau\tau} + 2\frac{M_\infty^2}{l}\varphi_{x\tau} - \left[\left(1 - M_\infty^2\right)\varphi_{xx} + \varphi_{yy} + \varphi_{zz}\right]\right\}\mathrm{d}\Omega = 0 \tag{7}$$

where W is an appropriate set of weighting functions and Ω is the space domain of interest. Clearly Ω does not extend to infinity, so that Eq. (5b) is approximately enforced at a finite distance. This deserves further comment but is not of interest here inasmuch as the conclusions to be drawn will not change. To lower the order of differentiation with respect to the space variables, the usual integration by parts is carried out onto the terms having second-order derivatives in space, yielding

$$\int_\Omega W^T\left[\left(\frac{M_\infty}{l}\right)^2\varphi_{\tau\tau} + 2\frac{M_\infty^2}{l}\varphi_{x\tau}\right]\mathrm{d}\Omega + \int_\Omega\left[W_x^T\left(1 - M_\infty^2\right)\varphi_x + W_y^T\varphi_y + W_z^T\varphi_z\right]\mathrm{d}\Omega + \int_\sigma W^T\varphi_n\,\mathrm{d}\sigma = 0 \tag{8}$$

Note that in deriving Eq. (8) the slenderness condition has been taken into account so that φ_n is fully determined by Eq. (6a), which appears as a natural boundary term. By adopting the same weighting, the Kutta condition also can be weakly enforced, which yields

$$\int_{\sigma_w} W^T\left(\Delta\varphi_x + \frac{1}{l}\Delta\varphi_\tau\right)\mathrm{d}\sigma_w = 0 \tag{9}$$

The actual finite element discretization can be carried out once a mesh geometry is specified so that the potential φ and the weight can be approximated by appropriate nodal shape functions N_i and W_i. Thus, if x is the generic point and n the total number of nodes, the finite elements approximation reads

$$\varphi(x, \tau) = \Phi(x)\phi(\tau) \tag{10}$$

where the nodal parameters ϕ_i, organized in the vector ϕ, become the primary unknowns. Using Eq. (10), Eqs. (8) and (9) become

$$F\phi'' + G\phi' + H\phi = B_q q + B_{\dot{q}}q' + B_g(v_g/V_\infty) \tag{11}$$

$$S\phi' + T\phi = \mathbf{0} \tag{12}$$

whose matrices, all depending on the freestream Mach number and reference trim condition, can be easily inferred from Eqs. (8) and (9) without any further definition.

Along with the numerical solution of Eq. (12), the generalized aerodynamic forces can be computed with

$$f_a = \int_\sigma (Q \cdot \boldsymbol{n})^T c_p \, \mathrm{d}\sigma = -2 \int_\sigma (Q \cdot \boldsymbol{n})^T \left(\varphi_x + \frac{1}{l}\varphi_\tau \right) \mathrm{d}\sigma \quad (13)$$

where $\boldsymbol{n}$ is the unit vector normal to the nondeformed surface of the body and leads to the discretized form

$$\boldsymbol{f}_a = C_\varphi \phi + C_{\dot\varphi} \phi' \quad (14)$$

Before viewing Eqs. (11) and (14) as a finite state approximation of the aerodynamics, the constraint, i.e., Eq. (12), must be eliminated. To this end we choose an independent set f and a wake-dependent set w of equations and variables and partition all of the terms accordingly, i.e.,

$$\phi = \begin{Bmatrix} \phi_f \\ \phi_w \end{Bmatrix}, \qquad F = \begin{bmatrix} F_{ff} & F_{fw} \\ F_{wf} & F_{ww} \end{bmatrix}, \qquad G = \begin{bmatrix} G_{ff} & G_{fw} \\ G_{wf} & G_{ww} \end{bmatrix}$$

$$H = \begin{bmatrix} H_{ff} & H_{fw} \\ H_{wf} & H_{ww} \end{bmatrix}, \qquad B_q = \begin{Bmatrix} B_{fq} \\ B_{fq} \end{Bmatrix}, \qquad B_{\dot q} = \begin{Bmatrix} B_{f\dot q} \\ B_{w\dot q} \end{Bmatrix}$$

$$B_{v_g} = \begin{Bmatrix} B_{fg} \\ B_{fg} \end{Bmatrix}, \qquad S = \lfloor S_f \quad S_w \rfloor, \qquad T = \lfloor T_f \quad T_w \rfloor$$

We then write Eq. (12) as $S_w \phi'_w + S_f \phi'_f + T_w \phi_w + T_f \phi_f = 0$, from which

$$\phi'_w = -S_w^{-1}(S_f \phi'_f + T_w \phi_w + T_f \phi_f) \quad (15a)$$

Differentiating Eq. (15a) with respect to τ, we obtain

$$\phi''_w = -S_w^{-1}(S_f \phi''_f + T_w \phi'_w + T_f \phi'_f) \quad (15b)$$

and, finally, after substituting ϕ'_w obtained from Eq. (15a) into Eq. (15b), we have

$$\phi''_w = -S_w^{-1}\left[S_f \phi''_f + \left(T_f - T_w S_w^{-1} S_f\right)\phi'_f - T_w S_w^{-1}(T_w \phi_w + T_f \phi_f)\right] \quad (16)$$

Using the preceding ϕ'_w and ϕ''_w in the partitioned Eq. (11), we can write

$$\bar F_{ff} \phi''_f + \bar G_{ff} \phi'_f + \bar H_{ff} \phi_f + \bar H_{fw} \phi_w = B_{fq} \boldsymbol{q} + B_{f\dot q} \boldsymbol{q}' + B_{fg}(\boldsymbol{v}_g / V_\infty) \quad (17a)$$

$$\bar F_{wf} \phi''_f + \bar G_{wf} \phi'_f + \bar H_{wf} \phi_f + \bar H_{ww} \phi_w = B_{wq} \boldsymbol{q} + B_{w\dot q} \boldsymbol{q}' + B_{wg}(\boldsymbol{v}_g / V_\infty) \quad (17b)$$

so that, after evaluating ϕ_w from Eq. (17b) and substituting it into Eq. (17a), we obtain the following system of second-order ordinary differential equations in ϕ_f:

$$\bar F \phi''_f + \bar G \phi'_f + \bar H \phi_f = \bar B_q \boldsymbol{q} + \bar B_{\dot q} \boldsymbol{q}' + \bar B_g (\boldsymbol{v}_g / V_\infty) \quad (18)$$

Without going into the details, the same procedure can be trivially applied to Eq. (14) so that the generalized aerodynamic forces can be set finally in the following form:

$$\boldsymbol{f}_a = \bar C_\varphi \phi_f + \bar C_{\dot\varphi} \phi'_f + \bar D_q \boldsymbol{q} + \bar{\boldsymbol{D}}_{\dot q} \boldsymbol{q}' + \bar D_g (\boldsymbol{v}_g / V_\infty) \quad (19)$$

to which we can associate the following linear finite states model of the aerodynamics:

$$\boldsymbol{x}'_a = A_a \boldsymbol{x}_a + B_q \boldsymbol{q} + B_g (\boldsymbol{v}_g / V_\infty) \quad (20a)$$

$$\boldsymbol{f}_a = C_a \boldsymbol{x}_a + D_q \boldsymbol{q} + D_{\dot q} \boldsymbol{q}' + D_g (\boldsymbol{v}_g / V_\infty) \quad (20b)$$

where

$$\boldsymbol{x}_a = \begin{Bmatrix} \phi_f \\ \psi_f \end{Bmatrix}, \qquad A_a = \begin{bmatrix} -\bar F^{-1} \bar G & \bar F^{-1} \\ -\bar H & 0 \end{bmatrix}, \qquad B_q = \begin{Bmatrix} \bar F^{-1} \bar B_q \\ \bar B_{\dot q} \end{Bmatrix}$$

$$B_g = \begin{Bmatrix} 0 \\ \bar B_g \end{Bmatrix}, \qquad C_a = \lfloor \bar C_\varphi - \bar C_{\dot\varphi} \bar F^{-1} \bar G \quad \bar C_{\dot\varphi} \bar F^{-1} \rfloor$$

and $D_q = \bar D_q$, $D_{\dot q} = \bar D_{\dot q}$, and $D_g = \bar D_g$.

The steps and the definitions required to obtain the output equation (20b) are not essential and can be easily worked out. Equations (20) show that a linearized approximation of the aerodynamics can be cast into a single finite state form so that different state approximations for the input related to the structural motions and gusts are not required. Moreover, because the aerodynamics is asymptotically stable, all of the eigenvalues of the matrix A_a must have a negative real part with a consequent exponential decay of the related impulse responses. This can appear in contrast to the well-established fact that the transfer function associated with small potential perturbations presents a $(jk)^k \log(jk)$ singularity at zero reduced frequency, which corresponds to a time decay of the type $1/\tau^k$ (Refs. 1 and 15). This contradiction can be associated to the truncation of the solution domain implied in Eq. (20), so that the preceding singularity cannot be resolved in transfer functions that are approximated by a numerical solution onto a truncated domain. Equations (20) can involve hundreds of thousands of states and have rarely been worked out in the given form. However, some relatively recent studies aim at supporting the idea that a similar approach is feasible even for complex CFD applications.[16] Note, however, that the aerodynamic eigensolutions must be derived from discretized formulations that have none of the nice features, e.g., symmetry with the consequent real eigensolutions and guaranteed diagonalization, that have made model order reduction through modal condensation a standard approach in structural analysis. Thus, for the moment, the use of aerodynamic modes is an interesting theoretical speculation that can be pursued for some simplistic models but is believed to be totally impractical for real applications. Furthermore, many de facto standard aerodynamic formulations used in aeroelastic analyses are based on the solution of integral equations in the frequency domain and completely hide any state structure of the aerodynamic.

Thus, as already stated, the formulation is used here just as a strongly needed, sound hint at a plausible state structure hidden behind the more commonly available aerodynamic transfer matrices defining the input–output relation associated with Eq. (20) in the reduced complex frequency domain p, i.e.,

$$\boldsymbol{f}_a = C_a (pI - A_a)^{-1} \left(B_q q + B_g \frac{\boldsymbol{v}_g}{V_\infty} \right) + (D_q + p D_{\dot q}) \boldsymbol{q} + D_g \frac{\boldsymbol{v}_g}{V_\infty} = [H_{am}(p, M_\infty) \quad H_{ag}(p, M_\infty)] \begin{Bmatrix} \boldsymbol{q} \\ \dfrac{\boldsymbol{v}_g}{V_\infty} \end{Bmatrix} \quad (21)$$

where $p = (\sigma + j\omega) l / V_\infty = h + jk$. However, both $H_{am}(p, M_\infty)$ and $H_{ag}(p, M_\infty)$ are more commonly available, for a reference trim condition, at a discrete set of harmonic, i.e., for $p = jk$, reduced frequencies and Mach numbers. For simplicity, the same symbols are used for $\boldsymbol{f}_a$ and $\boldsymbol{q}$ in both the time and complex reduced-frequency domain, and no confusion should arise. Within such a framework, the problem becomes that of identifying, according to some optimality criteria, a stable finite states realization of a transfer matrix fitting the available data with the lowest possible number of states. Sometimes the term *minimum states approximation*[9–12] is used to emphasize the need for such a precise low-order model, but there is clearly no minimum order as the optimal solution is somewhat subjective and related to the criteria used to determine the best identification.

As has been noted, the identification is generally tried separately by defining two different models of the type

$$H_{am} = C_{am}(pI - A_{am})^{-1}B_{am} + D_{0m} + pD_{1m} + p^2D_{2m} \quad (22)$$

$$H_{ag} = C_{ag}(pI - A_{ag})^{-1}B_{ag} + D_{0g} + pD_{1g} + p^2D_{2g} \quad (23)$$

whereas a distinctive feature of the present work is the use of a unified approach, so that we write

$$H_a = [H_{am} \quad H_{ag}] = C_a(M_\infty)[pI - A_a(M_\infty)]^{-1}B_a(M_\infty) + D_{0a}(M_\infty) + pD_{1a}(M_\infty) + p^2D_{2a}(M_\infty) \quad (24)$$

where the Mach dependence is explicitly recalled and, except for C_a and A_a, all of the matrices imply a partition of the type $B_a = [B_{am} \quad B_{ag}]$, $D_a = \lfloor D_m \quad D_g \rfloor$.

It is important to note that Eq. (21) shows that separate identifications for the motion and gust transfer matrices are not justified by the theory. Thus, the use of Eqs. (22) and (23) is, in principle, wrong because it introduces different dynamics, along with the related states, for a single physical system, so that they can be accepted only as a trick to get better analytical interpolations for the available discrete numerical data. In this view, the difference between using Eqs. (22) and (23) and Eq. (24) is believed to be fundamental and not merely a matter of choosing between different parameterizations for the same dynamic system. However, we must then note that, although Eq. (24) avoids such a basic fault, it is still somewhat different from Eq. (21) in extending the polynomial part to p^2 for both the motion and gust terms. That must be justified and put into a correct relation to Eq. (21); otherwise some incongruities can arise from its use, especially in connection with the need for introducing time derivatives for the gust term that does not appear in Eq. (21). Thanks to the use of a single-state dynamics, a consistent justification can be given by remarking that, both for computational reasons related to the cost of approximating the high-frequency contents and because often they are not of interest for the problem at hand, the best fit of H_a is generally determined on the basis of a limited set of transfer matrices evaluated at relatively low reduced harmonic frequencies. Thus, the states related to eigenvalues of A_a far exceeding the maximum reduced frequency at which H_a is available are bound to be somewhat meaningless. This sets a computationally convenient natural limit onto the number of states that should be used but can lead to an unsatisfactory fit. Then the improper, i.e., polynomial, terms of Eq. (24) can be of help as, contrary to their appearance, which make them more and more meaningful as the frequency increases, such terms are physically justifiable as a low-frequency approximation of the high-frequency content of the aerodynamics.[17]

In fact it is always possible to choose an aerodynamic state vector that can be partitioned into slow and fast parts in such a way that the two are uncoupled, For example, we can, in principle, compute the principal vectors of Eq. (20a) and determine a Jordan canonical form by using the related similarity transformation. Thus, Eq. (20a) can be conceived as rewritten in the following form:

$$\begin{Bmatrix} x'_{as} \\ x'_{af} \end{Bmatrix} = \begin{bmatrix} A_{as} & \mathbf{0} \\ \mathbf{0} & A_{af} \end{bmatrix} \begin{Bmatrix} x_{as} \\ x_{af} \end{Bmatrix} + \begin{bmatrix} B_{qs} \\ B_{qf} \end{bmatrix} q + \begin{bmatrix} B_{gs} \\ B_{gf} \end{bmatrix} \frac{v_g}{V_\infty} \quad (25)$$

whose slow part can be taken as it is, whereas the fast part, to be approximated only at low frequencies, can be residualized dynamically. Such a residualization can be easily determined by taking the ith derivatives with respect to τ of the second part of Eq. (25):

$$x_{af}^{(i+1)} = A_{af}x_{af}^{(i)} + B_{qf}q^{(i)} + B_{gf}\left(v_g^{(i)}/V_\infty\right) \quad (26)$$

so that the fast states can be approximated in their low-frequency range by simply assuming that their derivatives from the $(i+1)$th one upward, ith-order residualization, are negligible. Thus, cascading Eq. (26) backward, we have

$$x_{af} = -\sum_{i=0}^{n} A_{af}^{-(i+1)}\left(B_{qf}q^{(i)} + B_{gf}\frac{v_g^{(i)}}{V_\infty}\right) \quad (27)$$

When Eq. (27), with $n = 2$, is substituted into Eq. (20b) and combined with the slow aerodynamics, discarding the indices related to the fast and slow partitions, we have

$$x'_a = A_a(M_\infty)x_a + B_a(M_\infty)\begin{Bmatrix} q \\ \dfrac{v_g}{V_\infty} \end{Bmatrix} \quad (28a)$$

$$f_a = C_a(M_\infty)x_a + D_{0a}(M_\infty)\begin{Bmatrix} q \\ \dfrac{v_g}{V_\infty} \end{Bmatrix} + D_{1a}(M_\infty)\begin{Bmatrix} q \\ \dfrac{v_g}{V_\infty} \end{Bmatrix}' + D_{2a}(M_\infty)\begin{Bmatrix} q \\ \dfrac{v_g}{V_\infty} \end{Bmatrix}'' \quad (28b)$$

which is the time equivalent of the transfer matrix of Eq. (24). The use of $n = 2$ is mainly determined by the fact that the accelerations are in the structural motion equations anyway. However, even if it improves the accuracy of the fit, $n = 2$ can create some problems in relation to the generalized forces related to gusts that will deserve further comment. A value of $n > 2$ is always inconvenient because it increases the number of states in multiples of the structural degrees of freedom (DOF) so that it is more effective to increase the order of x_a.

Note that, if no slow aerodynamic states are modeled, i.e., a complete residualization is carried out, we are led directly to a quasisteady aerodynamic approximation, which is simply a second-order power expansion of Eq. (21) around the null reduced frequency,[18] i.e.,

$$H_a = [H_{am} \quad H_{ag}] = D_{0a}(M_\infty) + pD_{1a}(M_\infty) + p^2D_{2a}(M_\infty) \quad (29)$$

with

$$D_{0a} = H_a(0) = \mathrm{Re}[H_a(0)] \quad (30a)$$

$$D_{1a} = \frac{\mathrm{d}H_a(0)}{\mathrm{d}p} = \frac{\mathrm{dIm}[H_a(0)]}{\mathrm{d}k} \quad (30b)$$

$$D_{2a} = \frac{\mathrm{d}^2\mathrm{Re}[H_a(0)]}{\mathrm{d}p^2} = \frac{\mathrm{d}^2\mathrm{Re}[H_a(0)]}{\mathrm{d}k^2} \quad (30c)$$

In relation to such a quasisteady approximation, we can develop a further physical interpretation of what is meant for residualization of a fast dynamics. In fact, recalling the relations between the aerodynamic pulse response matrix $P_a(\tau)$ and the corresponding transfer matrix, i.e.,

$$H_a(p) = \int_0^{+\infty} P_a(\tau)e^{-p\tau}\,\mathrm{d}\tau \quad (31a)$$

$$P_a(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H_a(k)e^{p\tau}\,\mathrm{d}p \quad (31b)$$

it can be verified that we can also write

$$D_{0a} = \int_0^{+\infty} P_a(\tau)\,\mathrm{d}\tau \quad (32a)$$

$$D_{1a} = -\int_0^{+\infty} P_a(\tau)\tau\,\mathrm{d}\tau \quad (32b)$$

$$D_{2a} = \int_0^{+\infty} P_a(\tau)\tau^2\,\mathrm{d}\tau \quad (32c)$$

On the other hand, we have

$$f_a(\tau) = \int_0^{+\infty} P_a(v)q(\tau - v)\,\mathrm{d}v \quad (33)$$

so if we assume that, because of the fast exponential decay of the impulse response of the aerodynamics, Eq. (33) can be evaluated by approximating $q(\tau)$ with

$$q(\tau - v) \cong q(\tau) + q'(\tau)(-v) + q''(\tau)v^2 \quad (34)$$

we obtain

$$f_a(\tau) = \left[\int_0^{+\infty} P_a(v)\,\mathrm{d}v\right]\boldsymbol{q}(\tau) + \left[-\int_0^{+\infty} P_a(v)v\,\mathrm{d}v\right]\boldsymbol{q}'(\tau) + \left[\int_0^{+\infty} P_a(v)v^2\,\mathrm{d}v\right]\boldsymbol{q}''(\tau) \tag{35}$$

which is exactly the time-domain correspondent of Eq. (29). Equation (35) synthesizes Eqs. (30) and (32) by establishing as fast that part of the aerodynamics having a pulse response that decays rapidly enough to ensure that a second-order power expansion of the input in the time domain is sufficient for a precise evaluation of any transient response. Note that in the low-frequency range Eq. (28) must be precise enough to contain the quasisteady approximation of Eq. (29) because this is extremely important for a correct comprehensive modeling of the whole dynamics of a deformable aircraft.

Single State-Space Numerical Approximation

Having proved that Eqs. (24) and (28) are suitable for a single identification of a finite states approximation of the aerodynamics from the knowledge of its transfer matrices, it remains to define a performance index for the best fit of the available data allowing the determination of the lowest-order finite states approximation possible under the constraint that the matrix A_a is stable. This introduces some arbitrariness in that different criteria are possible and, for an assigned criterion, the goodness of the fit improves as the number of states increases so that the definition of what is the best is somewhat subjective. Our approach is based on the following points:

1) There is a possibility of enriching the available transfer matrices with the use of a causal interpolation to ensure a smoother fit.[13,19]

2) The dimension and structure of the finite states model must be the same for all Mach numbers so that the related dependence can be explicitly recovered, a posteriori, by an interpolation of the matrices C_a, A_a, B_a, D_{0a}, D_{1a}, and D_{2a}.

3) A weighted minimization is required, for each Mach number, of the sum of the maximum singular values of the difference of the transfer matrices related to the discrete set of harmonic reduced frequencies, including the zero value, and the corresponding Eq. (24), i.e.,

$$\min_{(C_a,A_a,B_a,D_{0a},D_{1a},D_{2a})} \sum_{k=0,\dots,k_m} w(k)\sigma_{\max}\big[H_a(k) - C_a(jkI - A_a)^{-1}B_a + D_{0a} + jkD_{1a} + (jk)^2D_{2a}\big] \tag{36}$$

where $\sigma_{\max}(X)$ is the maximum singular value of a matrix X.

4) A diagonal matrix A_a with stable eigenvalues is required; complex conjugate eigenvalues are allowed so that the matrix is structured either with simple diagonal elements σ or 2×2 diagonal blocks of the type

$$\begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

the stability is guaranteed by the constraint $\sigma \le \bar{\sigma}$, with $\bar{\sigma} \le 0$ being assigned by the analyst.

5) Equation (24) must be a comprehensive model synthesizing all of the information contained in the original data; in particular, a pure quasisteady approximation, i.e., Eqs. (29) and (30), should be obtainable from its complete residualization. This implies that a very precise fit of the low-frequency data must be ensured.

6) The eigenvalues far exceeding the maximum reduced frequency used for the calculation of the aerodynamic transfer matrices should be residualized.

7) The order of the residualization for the gust transfer function should be freely chosen between one and two, allowing the analyst to trade the number of states with the use of dummy low-pass shaping filters, required to avoid possible inconsistencies related to the time differentiation of a particular gust time history or turbulence spectrum.

To satisfy the requirements related to point 5, Ref. 13 imposes that both $H_a(0)$ and $\mathrm{d}[H_a(0)]/\mathrm{d}p = \mathrm{dIm}[H_a(0)]/\mathrm{d}k$ are exactly satisfied, whereas the matching of $\mathrm{d}^2[H_a(0)]/\mathrm{d}p^2$ is not enforced. The use of such an approach has proved cumbersome, and successive experience has verified that a more viable solution comes from enforcing the match of a few transfer matrices, two or three, at frequencies closely spaced around zero by using large weights $w(k)$. That is much more flexible and, considering that the derivatives are evaluated numerically anyway, also allows a precise satisfaction of the second-order terms. However, because it reduces the number of unknowns without adding any further computational burden, the constraint on $H_a(0)$ is maintained exactly by setting

$$D_{0a} = H_a(0) - C_aA_a^{-1}B_a \tag{37}$$

Thus, Eq. (36) is changed to

$$\min_{(C_a,A_a,B_a,D_{1a},D_{2a})} \sum_{k=k_1,\dots,k_m} w(k)\sigma_{\max}\big\{[H_a(k) - H_a(0)] - C_a\big[(pI - A_a)^{-1} + A_a^{-1}\big]B_a + pD_{1a} + p^2D_{2a}\big\} \tag{38}$$

To clarify point 7, we must show how Eqs. (2) and (28) are combined to model the aeroelastic system. To this end we rewrite Eq. (28) in the physical time domain t, explicitly partitioning the terms related to structural motions and gusts:

$$\dot{\boldsymbol{x}}_a = (V_\infty/c)A_a\boldsymbol{x}_a + (V_\infty/c)B_{am}\boldsymbol{q} + (V_\infty/c)B_{ag}(\boldsymbol{v}_g/V_\infty) \tag{39}$$

$$\boldsymbol{f}_a = C_a\boldsymbol{x}_a + D_{0am}\boldsymbol{q} + (c/V_\infty)D_{1am}\dot{q} + (c/V_\infty)^2D_{2a}\ddot{\boldsymbol{q}} + D_{0ag}(\boldsymbol{v}_g/V_\infty) + (c/V_\infty)D_{1ag}(\dot{\boldsymbol{v}}_g/V_\infty) + (c/V_\infty)^2D_{2a}(\ddot{\boldsymbol{v}}_g/V_\infty) \tag{40}$$

We then substitute Eq. (40) into Eq. (2) and define the following: a state $\boldsymbol{r}$

$$\boldsymbol{r} = M_s\dot{\boldsymbol{q}} + C_s\boldsymbol{q} - q(c/V_\infty)^2D_{2am}\dot{\boldsymbol{q}} - q(c/V_\infty)D_{1am}\boldsymbol{q} - q(c/V_\infty)D_{1ag}(\boldsymbol{v}_g/V_\infty) - q(c/V_\infty)^2D_{2ag}(\dot{\boldsymbol{v}}_g/V_\infty) \tag{41}$$

an aeroelastic state $\boldsymbol{x}_{\mathrm{ae}}^T = [\boldsymbol{q}^T \;\; \boldsymbol{r}^T \;\; \boldsymbol{x}_a^T]$, and a gust input $g^T = 1/V_\infty[\boldsymbol{v}_g^T \;\; \dot{\boldsymbol{v}}_g^T]$. Then a descriptor form of the response equation for a linearized aeroelastic system can be written as

$$V_{\mathrm{ae}}\dot{\boldsymbol{x}}_{\mathrm{ae}} = A_{\mathrm{ae}}\boldsymbol{x}_{\mathrm{ae}} + B_f\boldsymbol{f} + B_g g \tag{42}$$

with the different terms given by the following:

$$V_{\mathrm{ae}} = \begin{bmatrix} M_{\mathrm{ae}} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad A_{\mathrm{ae}} = \begin{bmatrix} -C_{\mathrm{ae}} & I & 0 \\ -K_{\mathrm{ae}} & 0 & qC_a \\ (V_\infty/c)B_{am} & 0 & (V_\infty/c)A_a \end{bmatrix}$$

$$B_g = \begin{bmatrix} B_{0g} & B_{1g} \end{bmatrix} = \begin{bmatrix} q(c/V_\infty)D_{1ag} & q(c/V_\infty)^2D_{2ag} \\ qD_{ag} & 0 \\ (V_\infty/c)B_{ag} & 0 \end{bmatrix} \tag{43}$$

$$B_f = \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}$$

$$M_{\mathrm{ae}} = M_s - q(c/V_\infty)^2D_{2am}, \qquad C_{\mathrm{ae}} = C_s - q(c/V_\infty)D_{1am}$$

$$K_{\mathrm{ae}} = K_s - qD_{0am}$$

It can be seen that, with the preceding translation of the aeroelastic equations to a state form, the first derivative of the gust input is still required so that discontinuous, e.g., stepped, deterministic gusts and white noise approximations of a random turbulence cannot be taken into account directly. As stated in point 7, one can then revert to a first-order residualization for the gust input, but this will require more states for the same fit accuracy. Our preferred solution is to use a dummy low-pass filter and rewrite Eq. (42) as

$$\begin{bmatrix} V_{ae} & -B_{1g} \\ 0 & I \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}}_{ae} \\ \dot{\boldsymbol{x}}_f \end{Bmatrix} = \begin{bmatrix} A_{ae} & B_{0g} \\ 0 & V_f \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}_{ae} \\ \boldsymbol{x}_f \end{Bmatrix} + \begin{bmatrix} B_f \\ 0 \end{bmatrix} \boldsymbol{f} + \begin{bmatrix} B_{0g} \\ 0 \end{bmatrix} g \tag{44}$$

where V_f is a scalar matrix of the same order of g whose values define time constants faster than any of those of the aeroelastic system, so that the discontinuities are smoothed without significant changes in the response and the power spectral density of the white noise turbulence is maintained fairly constant over the system frequency response. Often this trick is not required, and a typical much used instance of such a case is a Dryden spectrum turbulence, to which we can associate the following shaping filter:

$$\frac{v_g}{V_\infty} = \left(\frac{\sigma_g}{V_\infty}\sqrt{\frac{L_g}{V_\infty}}\right)\left[\left(1 + s\sqrt{3\frac{L_g}{V_\infty}}\right) \bigg/ \left(1 + s\frac{L_g}{V_\infty}\right)^2\right] w \tag{45}$$

where σ_g is the turbulence intensity, L_g the turbulence reference length, and w a unit intensity white noise. This can then be directly accounted for in the following equation:

$$\begin{bmatrix} V_{ae} & -B_{1g} & 0 \\ 0 & \left(\frac{L_g}{V_\infty}\right)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}}_{ae} \\ \frac{\dot{v}_g}{V_\infty} \\ \dot{x}_g \end{Bmatrix} = \begin{bmatrix} A_{ae} & B_{0g} & 0 \\ 0 & -2\frac{L_g}{V_\infty} & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}}_{ae} \\ \frac{v_g}{V_\infty} \\ x_g \end{Bmatrix}$$

$$+ \left(\frac{\sigma_g}{V_\infty}\sqrt{\frac{L_g}{V_\infty}}\right) \begin{bmatrix} 0 \\ \sqrt{3\frac{L_g}{V_\infty}} \\ 1 \end{bmatrix} w + \begin{bmatrix} B_f \\ 0 \\ 0 \end{bmatrix} \boldsymbol{f} \tag{46}$$

The solution of the nonlinear optimization points under the constraints set by points 4 and 5 is a formidable task, often involving many thousands of unknowns. However, it can be easily solved, even on today's personal computers, when a good starting solution and an efficient nonlinear optimization routine are provided. To this end we follow Ref. 13 and start with a left polynomial matrix approximation of H_a in the form

$$H_a(p) = \left(p^n + \sum_{i=0}^{n-1} D_i p^i\right)^{-1} \left(\sum_{i=0}^{n+2} N_i p^i\right) \tag{47}$$

Equations (39) and (40) can be put in the form of Eq. (24) through a polynomial division so that

$$H_a(p) = D_{0a} + pD_{1a} + p^2 D_{2a} + \left(p^n + \sum_{i=0}^{n-1} \bar{D}_i p^i\right)^{-1} \left(\sum_{i=0}^{n-1} \bar{R}_i p^i\right) \tag{48}$$

where $\bar{D}_i$ and $\bar{R}_i$ are the quotient and remainder terms related to Eq. (47). A full correspondence to Eq. (24) is then obtained by setting

$$C_a^T = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \qquad A_a = \begin{bmatrix} -\bar{D}_{n-2} & I & \dots & 0 \\ -\bar{D}_{n-2} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\bar{D}_1 & 0 & \dots & I \\ -\bar{D}_0 & 0 & \dots & 0 \end{bmatrix} \tag{48b}$$

$$B_a = \begin{bmatrix} \bar{R}_{n-1} \\ \bar{R}_{n-2} \\ \vdots \\ \bar{R}_1 \\ \bar{R}_0 \end{bmatrix}$$

The dimension of the finite states system of Eq. (40) depends on the order n of the polynomial approximation, and generally $n = 1$ is adequate. Note that the value of n is not critical because this is just a preliminary step aimed at setting an initial tentative solution, to be reduced and minimized subsequently.

The matrices D and N can be determined by casting Eq. (47) in the following form:

$$\sum_{i=0}^{n+2} (jk)^i N_i^T - H_a^T(jk) \sum_{i=0}^{n-1} (jk)^i D_i^T = (jk)^n H_a^T(jk) \tag{49}$$

that is successively written for each reduced harmonic frequency for which H_a has been computed, thus giving an overdetermined system of linear equations in the unknowns D and N, which is solved in a weighted least-square sense using the same weight to be adopted for Eq. (36), i.e.,

$$w(k_0)\left\{\sum_{i=1}^{n+2} (jk_0)^i N_i^T - H_a^T(jk_0) \sum_{i=1}^{n-1} (jk_0)^i D_i^T + \left[H_a^T(0) - H_a^T(jk_0)\right] D_0^T \right\} = w(k_0)\left[(jk_0)^n H_a^T(jk_0)\right]$$

$$w(k_1)\left\{\sum_{i=1}^{n+2} (jk_1)^i N_i^T - H_a^T(jk_1) \sum_{i=1}^{n-1} (jk_1)^i D_i^T + \left[H_a^T(0) - H_a^T(jk_0)\right] D_0^T \right\} = w(k_1)\left[(jk_1)^n H_a^T(jk_1)\right]$$

$$\vdots \tag{50}$$

$$w(k_{m-1})\left\{\sum_{i=0}^{n+2} (jk_{m-1})^i N_i^T - H_a^T(jk_{m-1}) \sum_{i=0}^{n-1} (jk_{m-1})^i D_i^T + \left[H_a^T(0) - H_a^T(jk_0)\right] D_0^T \right\} = w(k_{m-1})\left[(jk_{m-1})^n H_a^T(jk_{m-1})\right]$$

$$w(k_m)\left\{\sum_{i=0}^{n+2} (jk_m)^i N_i^T - H_a^T(jk_m) \sum_{i=0}^{n-1} (jk_m)^i D_i^T + \left[H_a^T(0) - H_a^T(jk_0)\right] D_0^T \right\} = w(k_m)\left[(jk_m)^n H_a^T(jk_m)\right]$$

Once again note that the zero-frequency constraint is exactly satisfied by imposing $N_0 = D_0 H_a(0)$.

The procedure is very fast, but the acquisition of a stable state matrix is not guaranteed. Usually the higher the order of the approximation, the easier it is to obtain an unstable matrix. Thus, the eigensolution of Eq. (44b) is determined and the corresponding state equation is set in the eigenvector space by a similarity transformation. Then criterion 6 is applied, and the real part of any unstable eigenvalue is set to $\bar{\sigma}$. This is done for all of the Mach numbers so that a state of the same order is ensured for all of the available data.

The solution thus obtained is then optimized by using a limited-memory, quasi-Newton minimization routine[20] that is capable of treating problems of large dimensions with side constraint on the variables (recall point 4), so that the stability can be maintained. Such a program seeks the optimum by directly solving Eq. (38). The variables involved in the minimization are, therefore, all of the elements of the matrices C_a, B_a, D_{1a}, and D_{2a} and the diagonal elements, or 2×2 blocks, of the matrix A_a. D_{0a} is recovered through Eq. (37) after completing the optimization.

The quasi-Newton minimization routine can use three optional methods: direct, dual, and conjugate gradient. All three formulations have been tried extensively in different numerical tests, and the method that demonstrated better overall performances for the problem at hand is the dual one. The direct method, although faster for each iteration step, requires many more iterations to reach convergence with a larger overall computational time, whereas in terms of computer time, the conjugate gradient was the least efficient of the three. The optimization program always has been powerful enough to improve the results decisively, even when starting from unstable polynomial approximations.

At this point, a single, very precise and stable finite states approximation, having the same order for each Mach number in the data set, is available so that the final interpolation of the coefficients of the matrices of Eqs. (39) and (40) can be carried out to allow the modeling for any operating condition of interest. Generally, the range of Mach numbers used is not extensive, and such an interpolation is easily determined by using low-order polynomials in M_∞.

The number of states of its proper part can now be decreased by using general-purpose and effective reduction methods[21,22] or, more simply, by residualizing those eigenvalues whose modules are much higher than the maximum reduced frequency of interest and then reoptimizing with the procedure explained earlier. Note that we prefer to run the numerical optimization also when the system reduction is determined on the basis of general-purpose reduction techniques because we always elect to use direct matching of the original data as the best-fit criterion.

Numerical Results

In this section two applications of the approach just proposed are presented. First, the method was tested on a two-DOF, two-dimensional, flat-plate Theodorsen formulation for an incompressible flow, including the related gust function as described by Sears (see Ref. 1). This remains a good test bench because, even if it cannot be resolved theoretically by a discretized set of data, the formulation contains the singularity $p^2 \log p$, associated to the Theodorsen function, that is somewhat difficult to approximate with exponential terms. This difficulty is strengthened by the need to match the gust transfer function within a single finite states approximation. In fact, the results reported in the literature[1] seem to indicate reasonably different time constants when the motion and the gust dependent terms are identified separately. Instead, the results shown in Figs. 1 and 2 demonstrate that it is possible to obtain an effective single finite states approximation over a fair range of reduced frequencies with only two states. (Note that, in all of the figures to be shown, the circle indicates the baseline data, along with the related harmonic reduced frequency, and the cross indicates the corresponding interpolated value, so that one can evaluate both the overall quality of the fit and the precision in matching the available data set.) These results were obtained by a preliminary matrix polynomial fit with $n = 1$, followed by the nonlinear minimization. Attempts for a further reduction to only one state, as well as to use a first-order residualization for the gust term, always led to similar, but much less precise, fits and revealed that the use of a second-order residualization for the gusts produces very significant improvements. The computer time taken from, and including, the rational polynomial approximation to the optimized two-state final model was about 3.5 min, regardless of the optimization option adopted. That, and all the computer time addressed in the following, is with the use of a 486DX2 66-MHz personal computer.

Second, the method was then applied to a 13×14 aerodynamic transfer matrix, with 13 DOF for the structural motions plus the vertical gust, related to the symmetric dynamic of the remotely piloted research vehicle (RPRV) shown in Fig. 3. The data set to be approximated is related to a zero Mach number and has been obtained by ALIS, a computer code using an analytically linearized subsonic unsteady potential formulation based on the Morino method.[23] Once again starting from a first-order polynomial approximation, i.e., 13 initial aerodynamic states, an excellent single identification has been achieved with only 7 states.

The computer times required to obtain the optimized 13-state identification was about 30 min using the dual approach and 45 min for the conjugate gradient formulation. The direct method required roughly the same time as the dual option, but, using the same convergence parameters, the quality of the fitting appeared poorer. That can be an indication of a relatively flat minimum and a confirmation of a certain arbitrariness, as mentioned at the beginning of the paper,

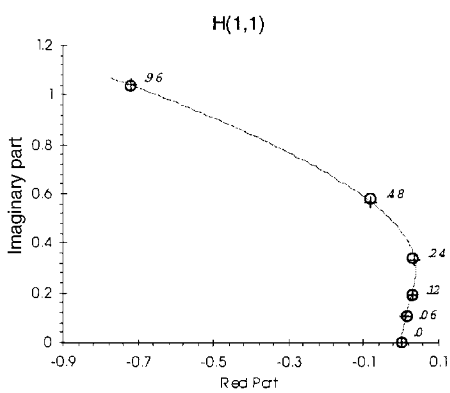

Fig. 1 Two-aerodynamic-state approximation of Theodorsen flat plate: input, plunge, and output, lift.

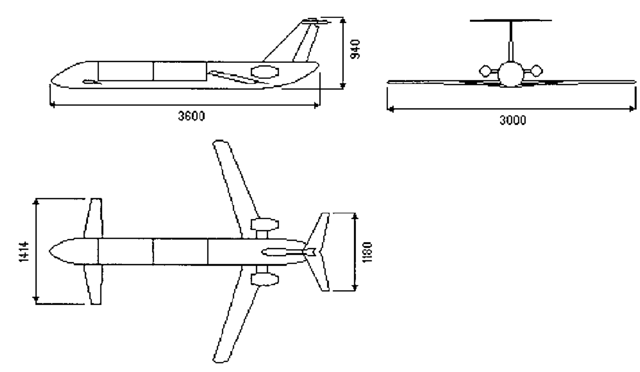

Fig. 3 RPRV.

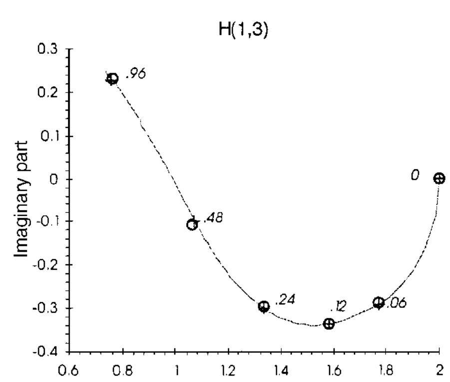

Fig. 2 Two-aerodynamic-state approximation of Theodorsen flat plate: input, gust, and output, lift (Sears gust function).

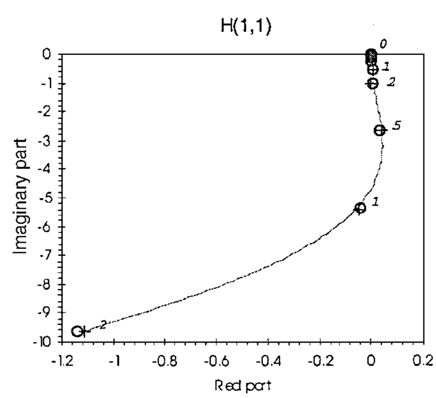

Fig. 4 RPRV: 13-DOF, seven-aerodynamic-state approximation: input, plunge, and output, lift.

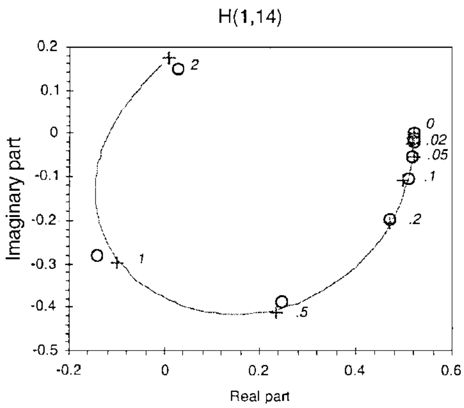

Fig. 5 RPRV: 13-DOF, seven-aerodynamic-state approximation: input, gust, and output, lift.

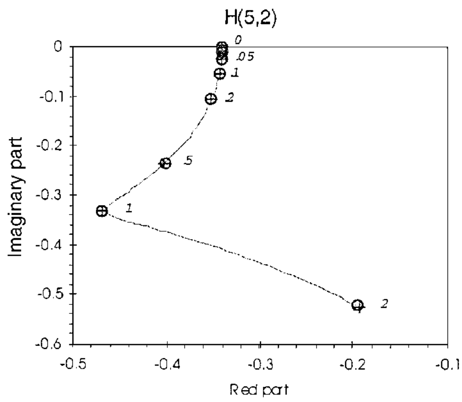

Fig. 6 RPRV: 13-DOF, seven-aerodynamic-state approximation: input, pitch, and output, generalized force on a high-frequency mode (mode 12).

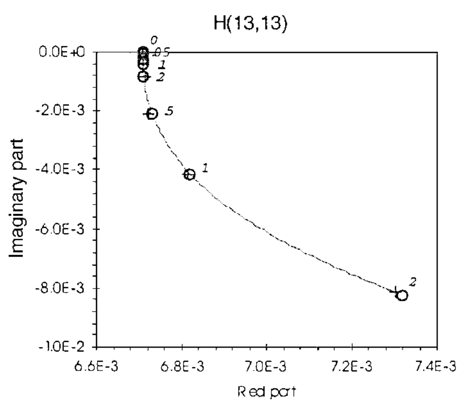

Fig. 7 RPRV: 13-DOF, seven-aerodynamic-state approximation: input, canard deflection, and output, canard hinge moment.

in defining the optimal best-fit criteria. In view of the performances, the optimized reduction from 13 to 7 states was carried out using the dual method and required about 10 min.

Some meaningful samples of the results obtained are presented in Figs. 4–7. Because the aeroelastic model had to be used to design an active control system integrating structural and flight controllers for stability augmentation, turbulence alleviation, and flutter suppression, the finite states approximation was required to approximate precisely the whole range of frequencies of interest from rigid-body modes to flutter. This emphasized the importance of constraining the fit at low reduced frequencies. Although the reduction from 13 to 7 states does not make much difference with respect to the whole aeroelastic model size, 36 states against 30, it allowed us to strain the verification of the level of accuracy required for such a wide frequency range of interest. In fact, in Fig. 8 one can see that, although the overall behavior is not changed significantly by the weighting, the low-frequency part is markedly affected, with profound effects on the rigid-body modes of the freely flying vehicle. Figure 9 shows that the use of the low-frequency weighted approximation leads to a consistent short-period frequency at different flight speeds, which compares well to the values calculated with standard flight-mechanics-type approximations of the aerodynamic derivatives,[24] whereas those related to the nonweighted approximation are not shown because their fit largely missed the correlation. It is important to note that the flutter condition was totally unaffected by the weighting. This and similar results obtained in other numerical verifications have permitted the demonstration of the viability, flexibility, and effectiveness of the use of weights in place of the exact imposition of the low-frequency constraints, as well as the importance of an adequate identification of the low-frequency behavior when the whole dynamics of the aircraft is of interest.

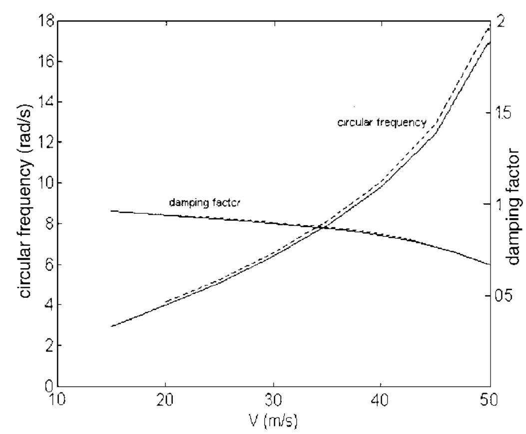

Fig. 9 Comparison between the short-period frequency and damping obtained by a standard flight mechanics method (. . . .) and the present method (——).

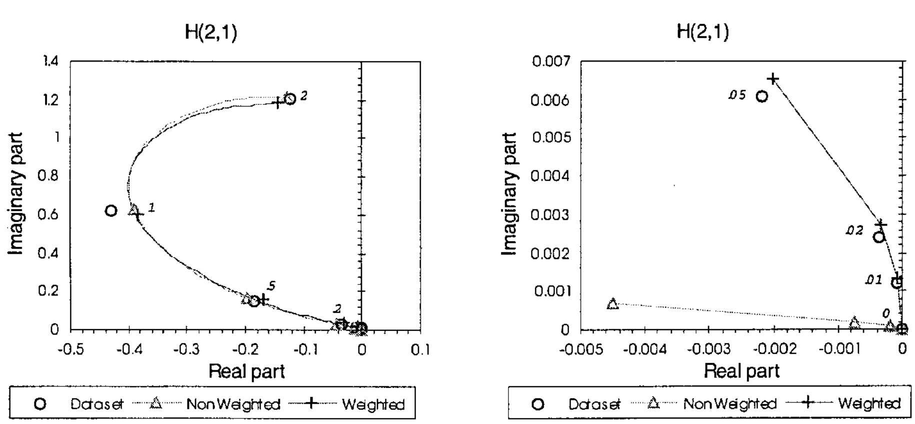

Fig. 8 RPRV: 13-DOF, seven-aerodynamic-state approximation; comparison of overall and detailed low-frequency weighted and unweighted approximations: input, plunge, and output, pitching moment.

Concluding Remarks

The paper has demonstrated the viability of using a single low-order, time-invariant, finite states dynamic system to approximate the linearized unsteady aerodynamic forces related to both structural motions and gusts. A numerical procedure, requiring the solution of a linear least-squares problem and a nonlinear large-order optimization, has been presented to allow a simple and efficient a posteriori identification of a reduced-order model from the related aerodynamic transfer matrices, available at a discrete set of harmonic reduced frequencies and Mach numbers. The same procedure can be used to refine and minimize the number of states required for an acceptable identification. Within such a framework, the importance of a precise matching of the low-frequency content of the aerodynamics has been emphasized in view of synthesizing a single comprehensive mathematical model, i.e., encompassing the whole vehicle dynamics from rigid-body modes to flutter, to be used for the analysis and design of aeroservoelastic systems in a modern approach.

References

[1]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison–Wesley, New York, 1955, Chaps. 5, 6.

[2]Fifer, S., *Analogue Computation: Theory, Techniques and Applications*, Vol. 4, McGraw–Hill, New York, 1961, Chap. 29.

[3]Richardson, J. R., "A More Realistic Method for Routine Flutter Calculations," *Proceedings of the AIAA Symposium on Structural Dynamics and Aeroelasticity*, AIAA, New York, 1965, pp. 10–17.

[4]Lanz, M., and Mantegazza, P., "Numerical Methods for Predicting the Aeroelastic Stability and Response of Flexible Airplanes," *Aerotecnica Missili e Spazio*, Vol. 63, No. 2, 1984, pp. 105–108.

[5]Lanz, M., and Mantegazza, P., "Modern Methods for the Analysis of the Dynamic Response of Aeroelastic Systems," *Aerotecnica Missili e Spazio*, Vol. 64, No. 3, 1985, pp. 170–176.

[6]Vepa, R., "On the Use of Padé Approximants to Represent Unsteady Aerodynamic Loads for Arbitrarily Small Motion of Wings," AIAA Paper 76-17, Jan. 1976.

[7]Roger, K. L., "Airplane Math Modeling Methods for Active Control Design," CP-228, AGARD, Aug. 1977.

[8]Edwards, J. W., Ashley, H., and Breakwell, J. V., "Unsteady Aerodynamic Modeling for Arbitrary Motions," *AIAA Journal*, Vol. 17, No. 4, 1979, pp. 365–374.

[9]Karpel, M., "Design for the Active Flutter Suppression and Gust Alleviation Using State-Space Aeroelastic Modeling," *Journal of Aircraft*, Vol. 19, No. 3, 1982, pp. 221–227.

[10]Karpel, M., "Reduced-Order Aeroelastic Models via Dynamic Residualization," *Journal of Aircraft*, Vol. 27, No. 5, 1990, pp. 449–455.

[11]Karpel, M., and Hoadley, S. T., "Physically Weighted Approximation of Unsteady Aerodynamic Forces Using the Minimum-State Method," NASA TP-3025, March 1991.

[12]Karpel, M., and Strul, E., "Minimum-State Unsteady Aerodynamic Approximations with Flexible Constraints," *Journal of Aircraft*, Vol. 33, No. 6, 1996, pp. 1190–1196.

[13]Ghiringhelli, G. L., and Mantegazza, P., "Interpolation Extrapolation and Modeling of Unsteady Linear(ized) Aerodynamic Forces," *Proceedings of the International Forum on Aeroelasticity and Structural Dynamics* (Strasbourg, France), Association Aéronautique et Astronautique de France, Paris, 1993, pp. 207–221.

[14]Morino, L., Mastrodotti, F., De Troia, R., Ghiringhelli, G. L., and Mantegazza, P., "Matrix Fraction Approach for Finite-State Aerodynamic Modeling," *AIAA Journal*, Vol. 33, No. 4, 1995, pp. 703–711.

[15]Milne, R. D., "Asymptotic Solutions of Linear Integro-Differential Equations," Ministry of Supply Repts. and Memoranda 3548, London, July 1966.

[16]Dowell, E. H., "Eigenmode Analysis in Unsteady Aerodynamics: Reduced-Order Models," *AIAA Journal*, Vol. 34, No. 8, 1996, pp. 1578–1583.

[17]Ghiringhelli, G. L., Lanz, M., Mantegazza, P., and Ricci, S., "Active Flutter Suppression Techniques in Aircraft Wings," *Integrated Technology Methods and Applications in Aerospace Systems Design*, edited by C. T. Leondes, Vol. 52, Control and Dynamics Series, Academic, New York, 1992, pp. 57–115.

[18]Rodden, W. P., and Giesing, J. P., "Application of Oscillatory Aerodynamic Theory to Estimation of Dynamic Stability Derivatives," *Journal of Aircraft*, Vol. 7, No. 3, 1970, pp. 272–275; "Errata and Addenda," *Journal of Aircraft*, Vol. 21, No. 1, 1984, pp. 93, 94.

[19]Mantegazza, P., "Interpolation and Extrapolation of Transfer Functions," *Aerotecnica Missili e Spazio*, Vol. 59, No. 1, 1980, pp. 69–75.

[20]Byrd, R. H., Lu, P., Nocedal, J., and Zhu, C., "A Limited Memory Algorithm for Bound Constrained Optimization," Dept. of Electrical Engineering and Computer Science, TR NAM-08, Northwestern Univ., Evanston, IL, May 1994.

[21]Chiang, R. Y., and Safonov, M. G., "Robust-Control Toolbox," MathWorks, Natick, MA, 1996, Chap. 1.

[22]Balas, G. J., Doyle, J. C., Glover, K., Packard, A., and Smith, R., "μ Analysis and Synthesis Toolbox," MathWorks, Natick, MA, 1995, Chap. 3.

[23]Bindolino, G. P., and Mantegazza, P., "Improvements on a Green's Function Method for the Solution of Linearized Unsteady Potential Flows," *Journal of Aircraft*, Vol. 24, No. 6, 1987, pp. 355–361.

[24]Roskam, J., *Airplane Design*, Univ. of Kansas, Lawrence, KS, 1989, Pt. 7.

A. Chattopadhyay
Associate Editor